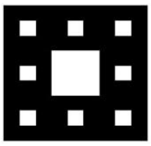
OBSEG PREPROGE SIERPINSKEGA

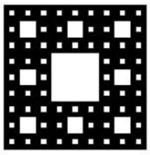
Osnovni element preproge Sierpinskega je kvadrat, za katerega vemo, da se obseg izračuna kot O$= 4 \times a$. Osnovni kvadrat S_0 ima osnovnico z dolžino 1.

$$O_0 = 4 \times 1 = 4$$

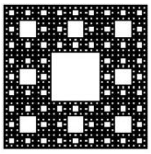
$$O_1 = 4 + 4 \times \frac{1}{3} = 4 + \frac{4}{3} = \frac{16}{3}$$



$$O_2 = 4 + \frac{4}{3} + 8 \times 4 \times \frac{1}{9} = \frac{80}{9}$$



$$O_3 = 4 + \frac{4}{3} + \frac{32}{9} + 8 \times 8 \times 4 \times \frac{1}{27} = \frac{496}{27}$$



$$O_n = 4 + \frac{4}{3} + \frac{32}{9} + \frac{256}{27} + \cdots + 8^{n-1} \times \frac{4}{3^n}$$

Splošno formulo O_n se preoblikuje.

$$\begin{aligned} O_n &= 4 + \frac{4}{3} + \frac{32}{9} + \frac{256}{27} + \cdots + \frac{4}{3} \times \left(\frac{8}{3}\right)^{n-1} = \\ &= 4 + \sum_{m=1}^{n} \frac{4}{3} \times \left(\frac{8}{3}\right)^{m-1} = \\ &= 4 + \frac{4}{3} \times \sum_{m=1}^{n} \left(\frac{8}{3}\right)^{m-1} \end{aligned}$$

Z indukcijo bomo sedaj pokazali, da je obseg množice O_n enaka $4 + \frac{4}{3} \times \sum_{m=1}^{n} \left(\frac{8}{3}\right)^{m-1}$. Videli smo že, da formula velja za $n = 0, 1, 2, 3$. Sedaj privzemimo, da formula velja za množico O_{n-1}, ter pokažimo, da potem velja tudi za množico O_n. Množico O_n dobimo iz množice O_{n-1}

tako, da ji prištejemo 8^{n-1} kvadratov s stranico dolžine $\frac{1}{3^n}$. Torej je obseg množice O_n enak obsegu množice O_{n-1} plus obseg odvzetih kvadratov. Torej:

$$
\begin{aligned}
O_n &= O_{n-1} + 8^{n-1} \times 4 \times \frac{1}{3^n} = \\
&= O_{n-1} + \frac{4}{3} \times \left(\frac{8}{3}\right)^{n-1} \\
&= 4 + \frac{4}{3} \times \sum_{m=1}^{n-1} \left(\frac{8}{3}\right)^{m-1} + \frac{4}{3} \times \left(\frac{8}{3}\right)^{n-1} = \\
&= 4 + \frac{4}{3} \times \left(\left(\frac{8}{3}\right)^{0} + \left(\frac{8}{3}\right)^{1} + \left(\frac{8}{3}\right)^{2} + \cdots + \left(\frac{8}{3}\right)^{(n-1)-1} + \left(\frac{8}{3}\right)^{n-1}\right) = \\
&= 4 + \frac{4}{3} \times \left(\left(\frac{8}{3}\right)^{0} + \left(\frac{8}{3}\right)^{1} + \left(\frac{8}{3}\right)^{2} + \cdots + \left(\frac{8}{3}\right)^{n-2} + \left(\frac{8}{3}\right)^{n-1}\right) = \\
&= 4 + \frac{4}{3} \times \sum_{m=1}^{n} \left(\frac{8}{3}\right)^{m-1}
\end{aligned}
$$